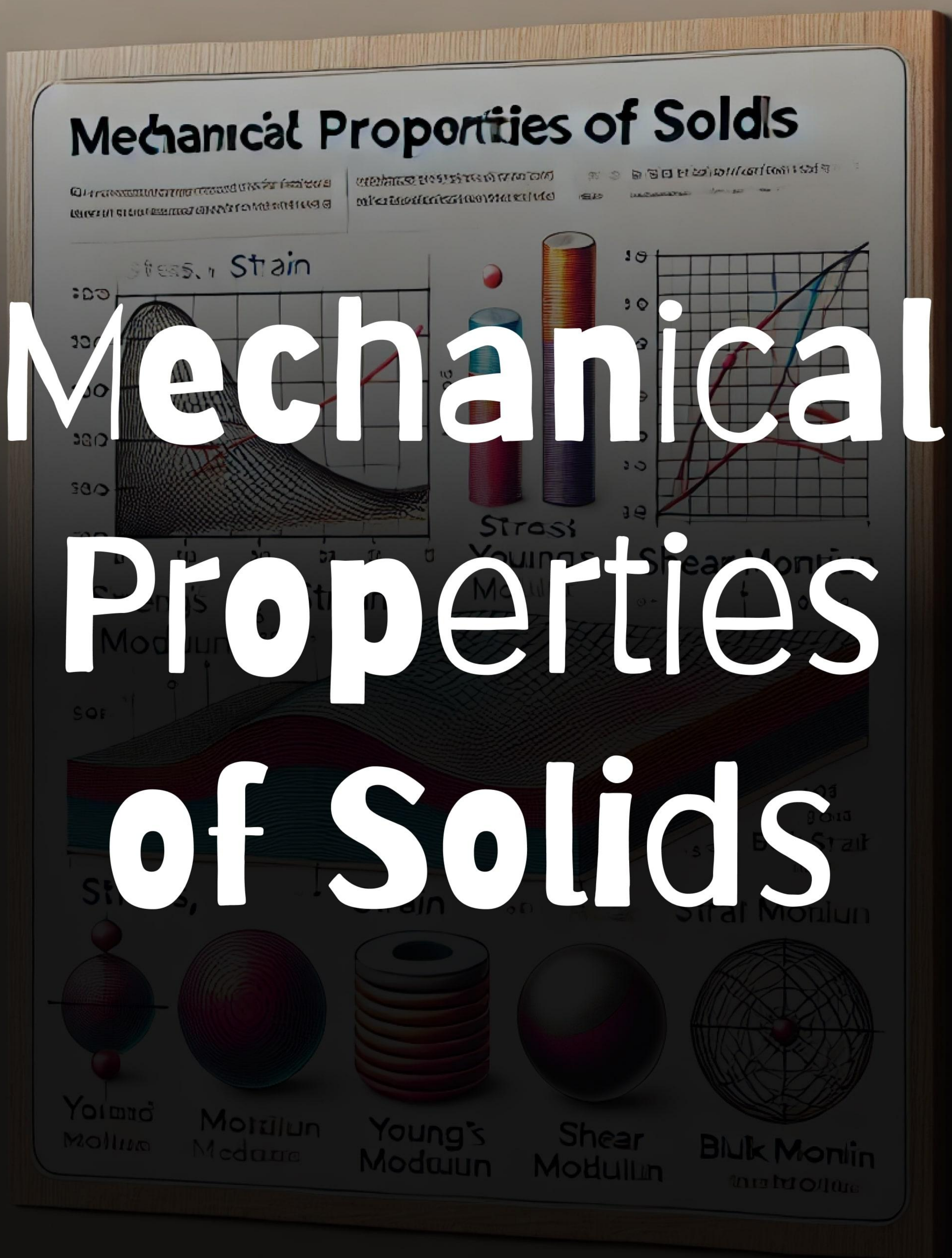
Mechanical Properties of Solids

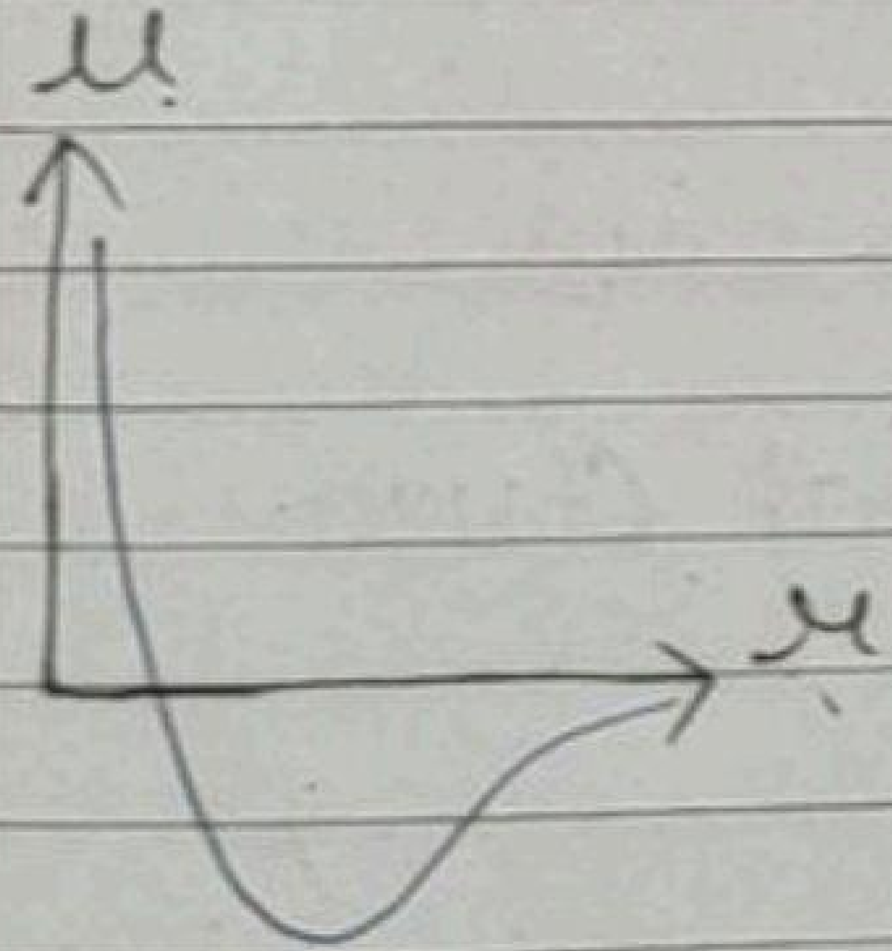


Mechanical Properties of Solids

Rigid body - A body in which distance b/w any two point remains same
→ No body is perfectly rigid

Elastic	Inelastic	Perfectly Inelastic (Plastic)
regain its shape after removing deforming force		

Restoring Force



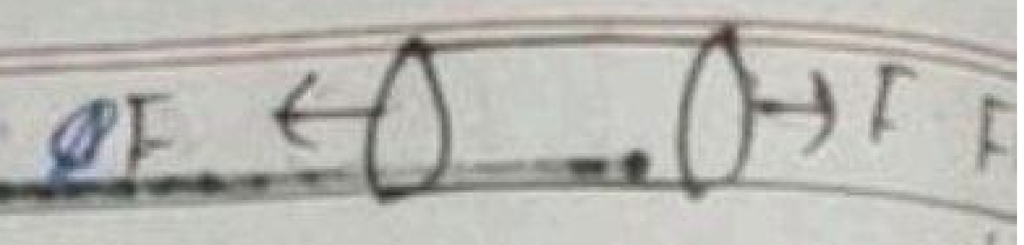
* $1\ dyne = 10^{-5}\ N$
$1\ cm^2 = 10^{-4}\ m^2$
$1\ mm = 10^{-3}\ m$
$1\ atm = 1.01 \times 10^{5}\ pa$

External Force

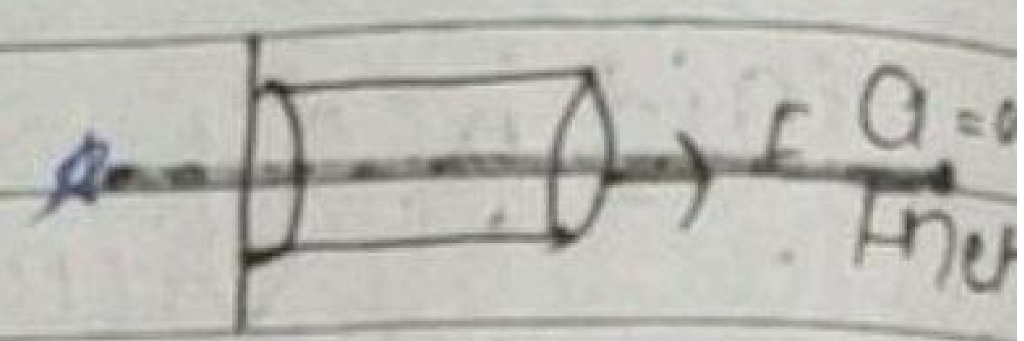
Compressive — Elongative/Tensile

$F_{net} = 2F$

If Net force in a body is zero then it can change its shape - Yes



Fnet = 23N
Tensile X
Compression ✓ = 3N



Identical body A & B then in which case Elongation?
⇒ Same in Both

Stress

(1) Normal / longitudinal stress
$= \frac{F_{elastic}}{Area}$

* N/m^2
* Scalar

(2) Shear stress / tangential stress
$= \frac{F_{external}}{Area}$

(3) Volumetric stress
$= \frac{F_{external}}{Area} = \Delta P$

Strain unit and dimensionless

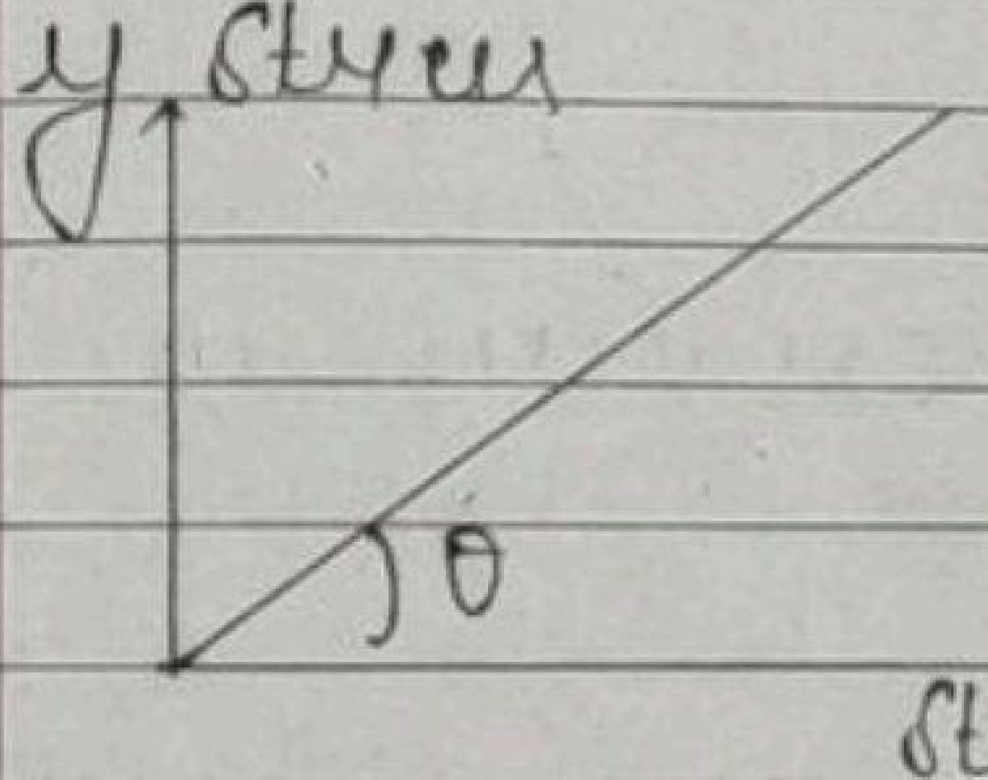
(1) Longitudinal — Shear — Volumeteric

$= \frac{\Delta l}{L_i} = \frac{L_f - L_i}{L_i}$; $\theta = \frac{\Delta l}{l_i}$; $\frac{\Delta V}{V_i}$

Hook's law

Stress $\propto$ Strain

Stress = Y Strain



★ $\tan\theta$ = Slope = Young modulus

Young modulus of Elasticity

Stress = Y Strain

$\frac{F}{A} = Y\frac{\Delta l}{l}$

$\frac{F}{\pi r^2} = Y\frac{\Delta l}{l} = Y\frac{l_f - l_i}{l}$

Coefficient of rigidity

$\frac{F}{A} = \eta\theta = \eta\frac{\Delta l}{l}$

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(3) **Bulk modulus of elasticity** -

$\Delta P \propto \frac{\Delta V}{V_i}$

$\Delta P = B \frac{\Delta V}{V_i}$

$B_{solid} > B_{liquid} > B_{gas}$

Imp. Points -

(1) Value of η, Y and B for perfectly elastic Body is Infinite.

(2) η and Y not defined for fluid and for gases their value is zero.

(3) $Y \propto \frac{1}{temperature} \propto \frac{1}{elongation}$

(4) Y may Increase or decrease on adding Impurities.

Q) The ratio of radii of two wires of same materials is 2 : 1. If these wires are stretched by equal force, the ratio of stresses produced

$Stress = \frac{F^{\nearrow same}}{Area} \propto \frac{1}{r^2}$

$\frac{S_1}{S_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 1 : 4

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Q) Two wire A and B are streched by same force, if for A and B, $Y_A : Y_B = 1 : 2$, $r_A : r_B = 3 : 1$ and $l_A : l_B = 4 : 1$, then ratio of their extension $\left(\frac{\Delta l_A}{\Delta l_B}\right)$ will be

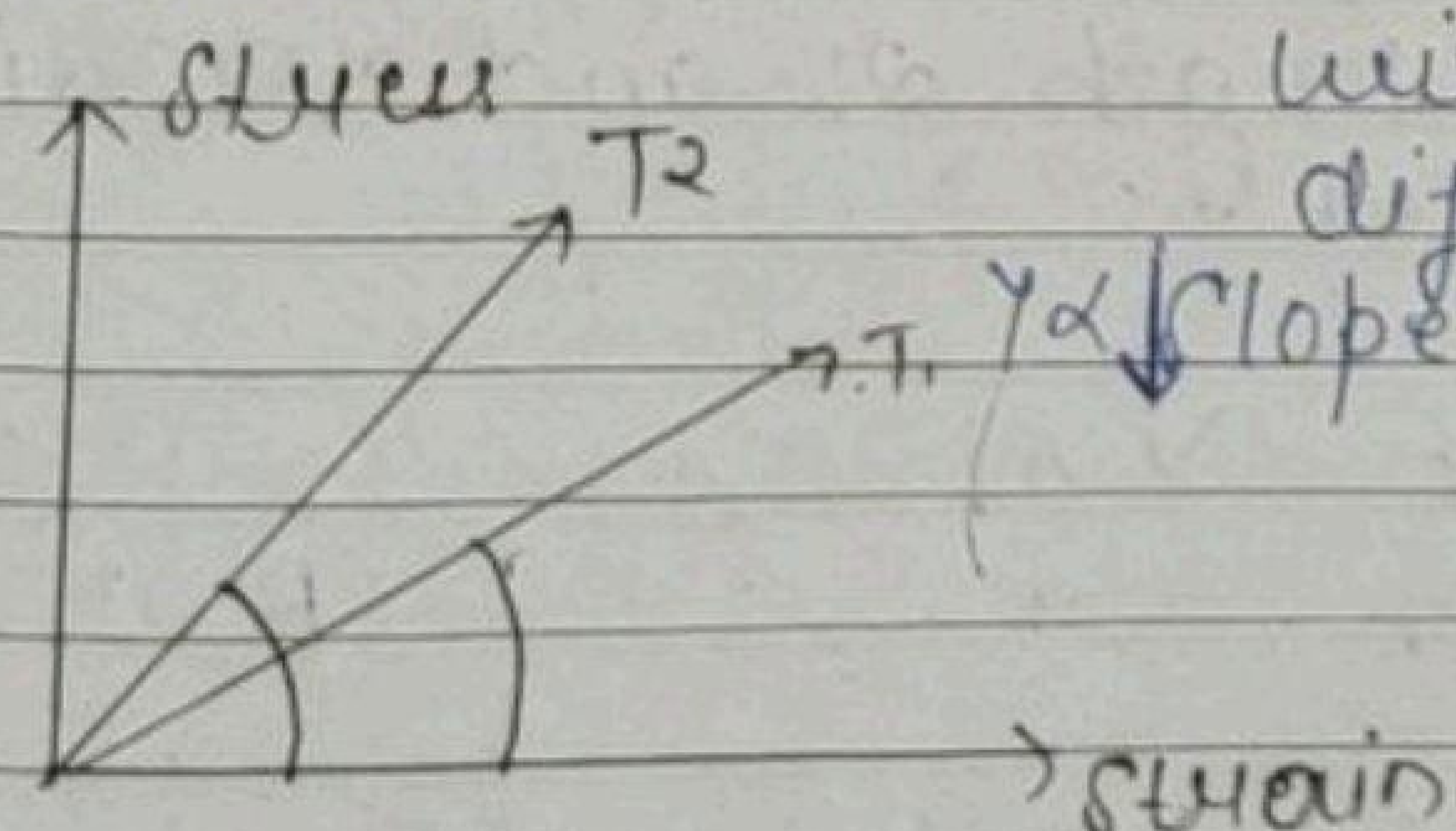
$\Delta l = \frac{Fl}{YB}$ $\quad \Delta l_{(A)} = \frac{4}{9}$ $\quad \Delta l_B = \frac{1}{2}$

$\frac{\Delta l_A}{\Delta l_B} = \frac{4}{9} \times \frac{2}{1} = 8/9$ $\quad 8 : 9$

Q) If in case A, elongation in wire of length L is l, then for same wire elongation in case B will be.

Same l

Q)



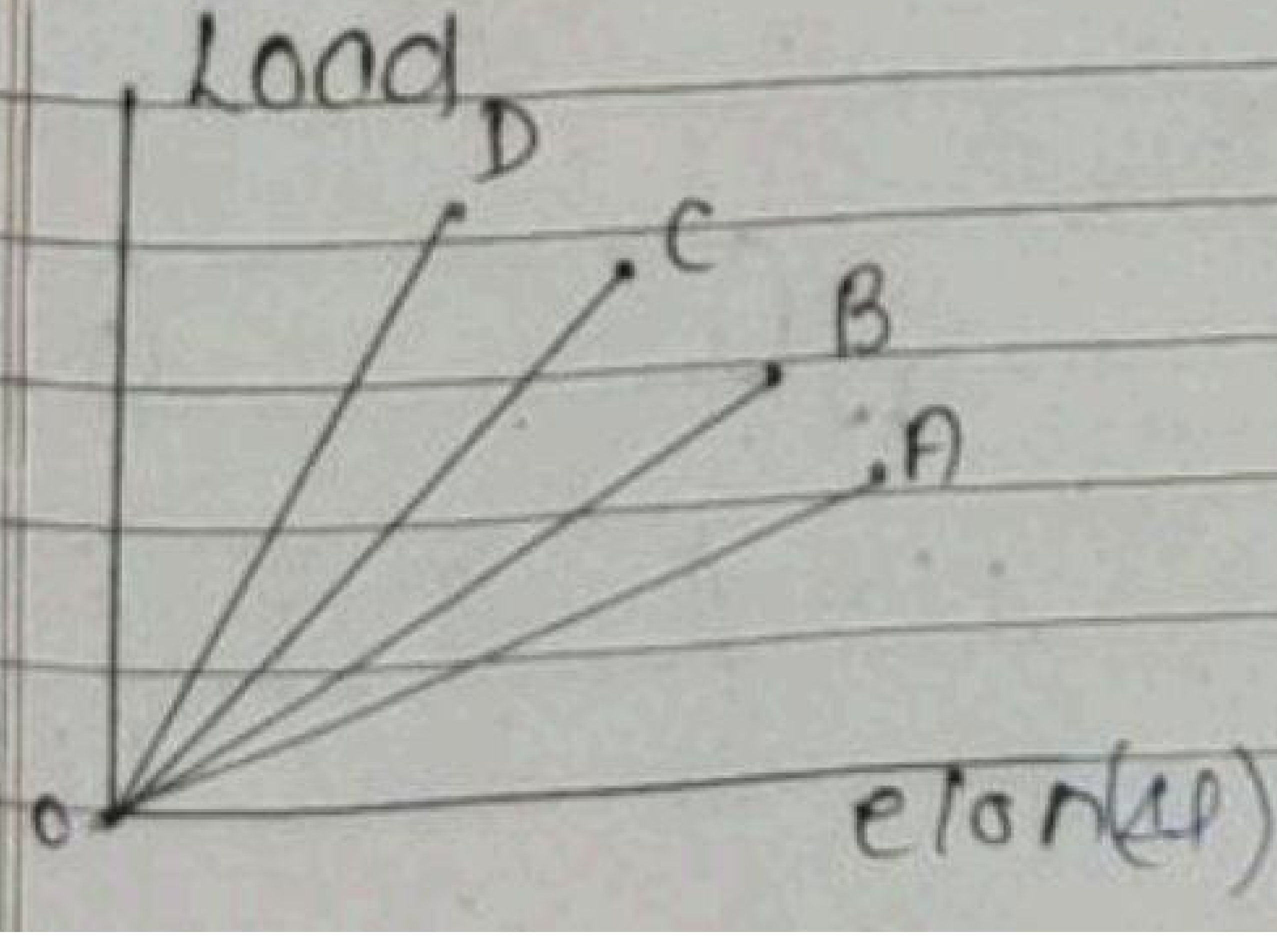
wire same at two different temperature

$Y \alpha \downarrow$ slope $\alpha \frac{1}{\text{Temp}\uparrow}$

slope $= Y \alpha \frac{1}{Te\uparrow}$

$T_1 > T_2$

Q)



l = same
Y = same

$\frac{F}{A} = \frac{Y \Delta l}{l}$ → sam.

$F = AY \Delta l$ depend

OA thinest

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Q) Two wires $\qquad F = A \, Y \frac{\Delta l}{l}$

Y - same

V - same $\qquad F = \frac{A \times A}{l\,A} = A^2$ $\quad \frac{}{\text{Volume same}}$

Δl bhi same lenge

$F \propto (\text{Area})^2 = (3F)^2 = 9F$

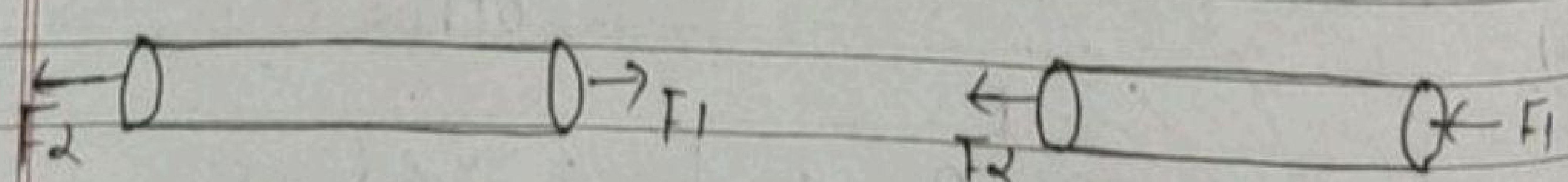
Q) Two wire A and B are stretched by same force if for A and B $Y_A : Y_B = 1:2$, $r_A : r_B = 3:1$ and $l_A : l_B = 4:1$ then r.

Q) A wire of length L and radius r fixed one end and a force F applied to other end produces an extension l. The ext. produced in another wire of same material of length 2L and radius 2r by a force 2F is

Initially - $\frac{F}{A} = Y \frac{\Delta l}{L}$ $\quad \Delta l = \frac{FL}{YA} = \frac{FL}{Y \pi r^2} = l$

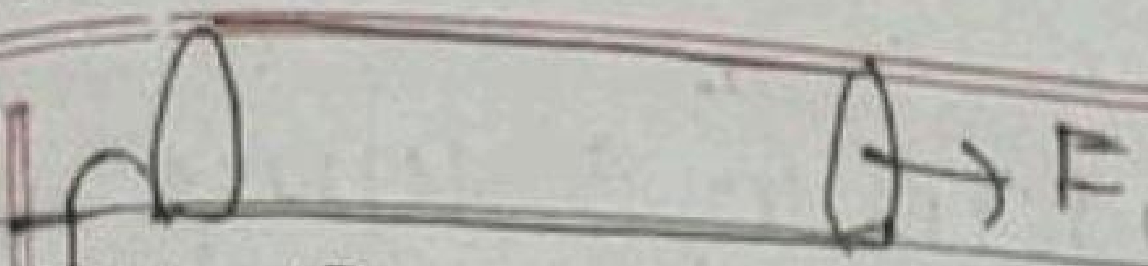
finally (l) = $\Delta L = \frac{2F \, 2L}{Y \pi (2r)^2} = \frac{2F \, 2L}{Y \, 4 \pi r^2} = \frac{FL}{Y \pi r^2}$

(7)



$\Delta L = \left(\frac{F_1 + F_2}{2}\right) \frac{l}{YA}$ $\qquad \Delta L = \left(\frac{F_1 - F_2}{2}\right) \frac{l}{YA}$

$d = \frac{m}{V}$

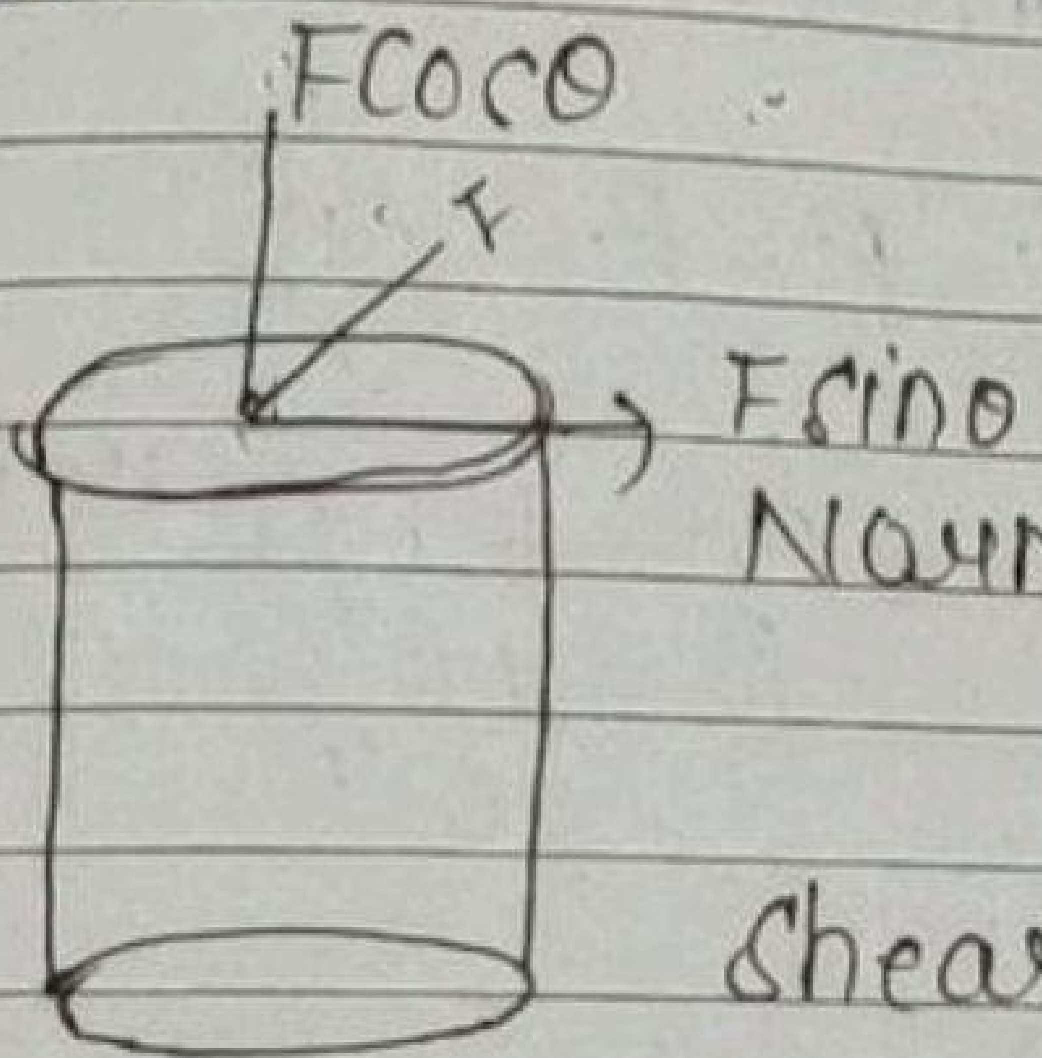


$\Delta L = \frac{Fl}{2YA}$

$\mu = 0$

$F_1 = F$

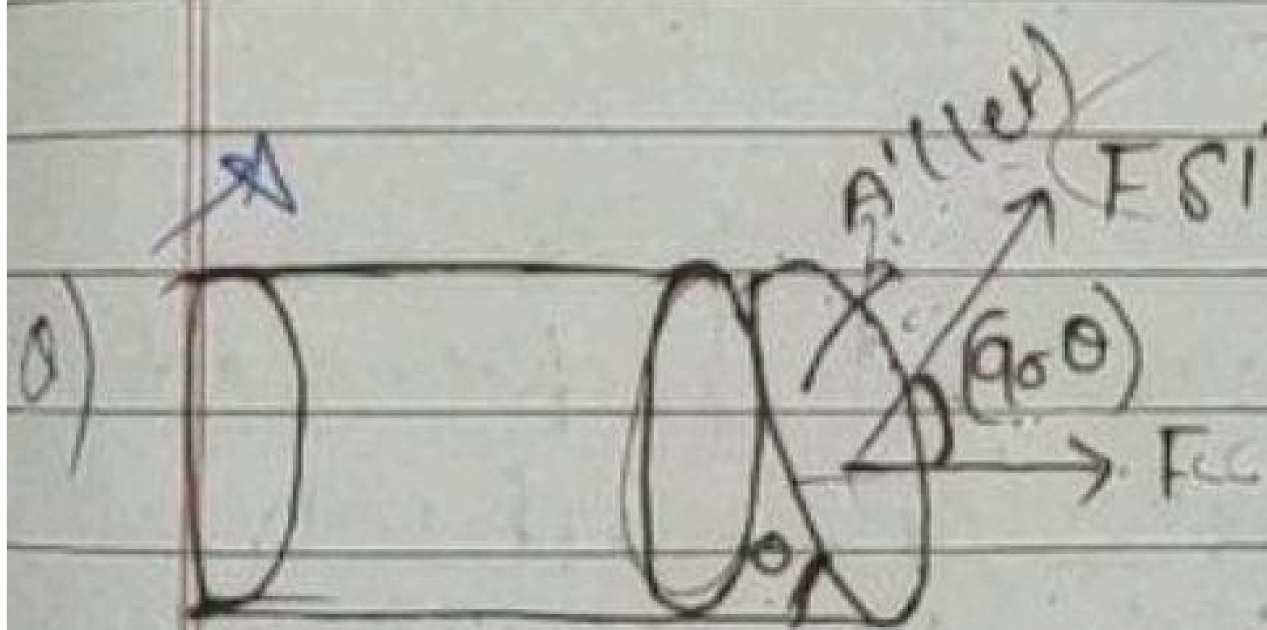
$F_2 = 0$



Normal Stress $= \frac{F\cos\theta}{A}$

Shear Stress $= \frac{F\sin\theta}{A}$

Q)



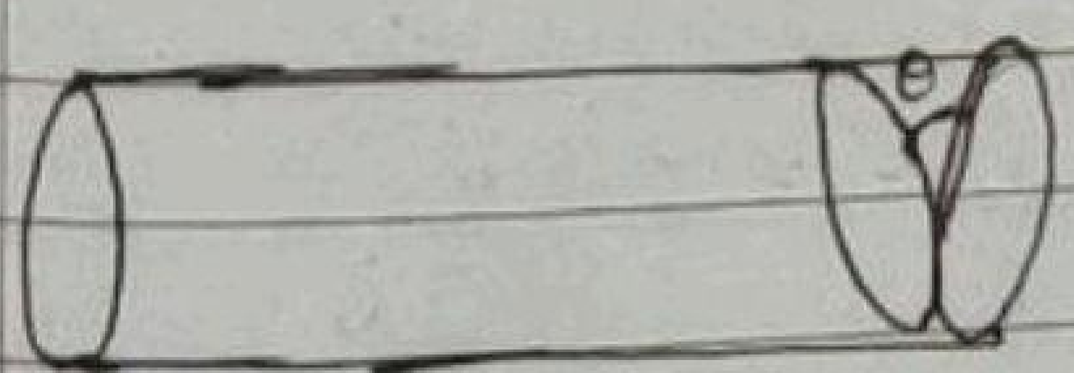
$F\sin\theta = F'$

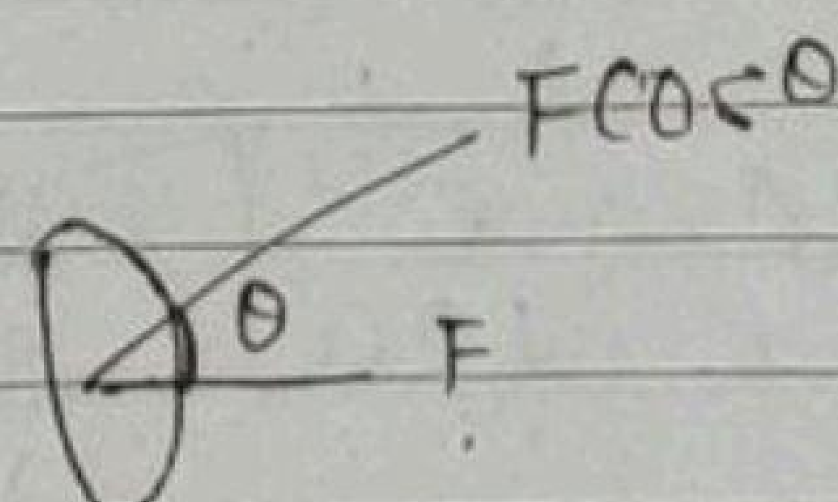
Normal Stress $= \frac{F'}{A'} = \frac{F\sin\theta}{\frac{A}{\sin\theta}}$

$= \frac{F\sin^2\theta}{A}$

$A'\sin\theta = A$

$A' = \frac{A}{\sin\theta}$





Stress $= \frac{F\cos\theta}{A'} = \frac{F\cos\theta}{\frac{A}{\cos\theta}} = \frac{F\cos^2\theta}{A}$

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Q) A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 100 atm. The bulk modulus of its material is.

1 atm = 1.01 × 10^5 Pa
100 atm = 100 × 10^5 Pa
= 10^7 Pa

$P = B\left(-\frac{\Delta V}{V}\right)$

$10^7 = B(10^{-4})$

$B = \frac{10^7}{10^{-4}} = 10^{7+4} = 10^{11}$

$\left[100 \times \frac{\Delta V}{V}\right] = 0.01$

$\frac{\Delta V}{V} = \frac{0.01}{100.00} = 10^{-4}$

Q) A uniform cubical block is subjected to volumetric compression, which decreases its each side by α%. The Bulk strain produces in it is.

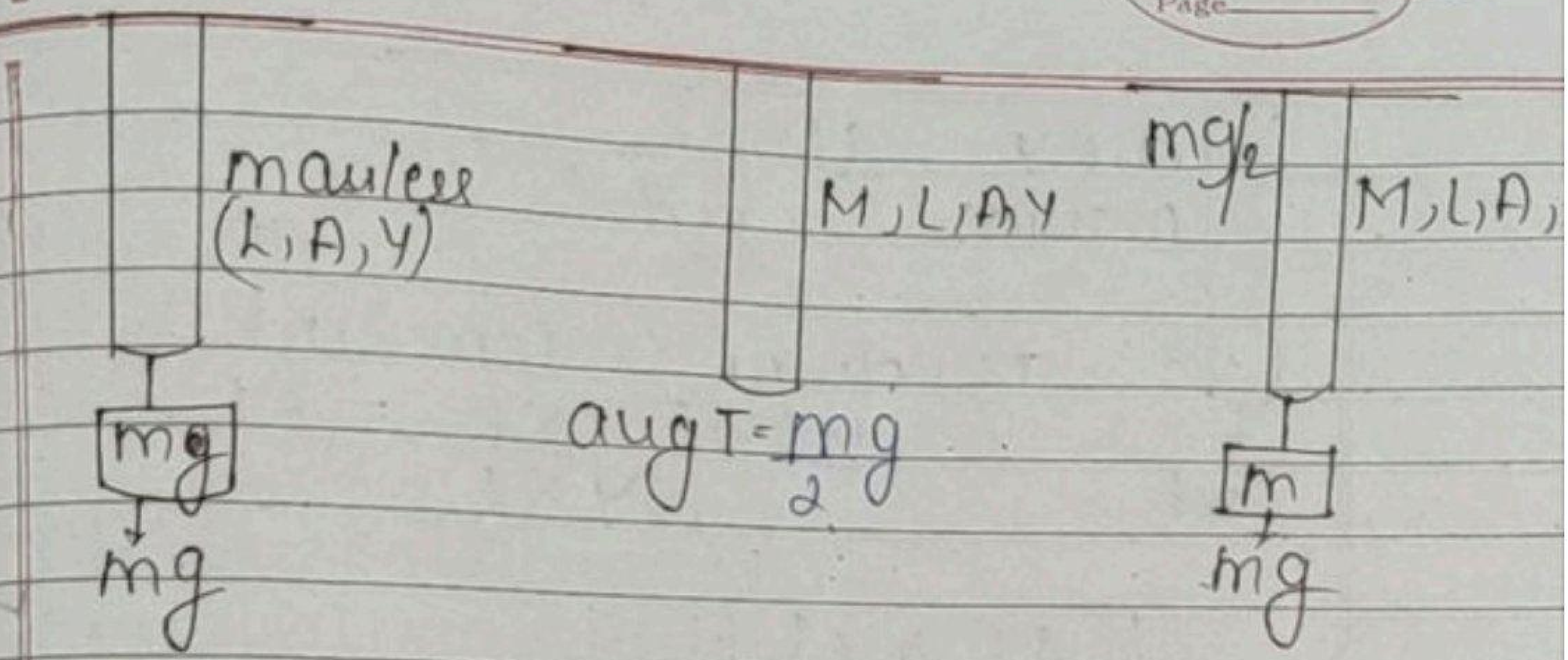
$100 \times \frac{\Delta l}{l} = \alpha\%$

$\left[\frac{\Delta l}{l} = \frac{\alpha}{100}\right]$

$V = l^3$

$\frac{\Delta V}{V} = 3\frac{\Delta l}{l} = \frac{3\alpha}{100} = 0.06$

* $B \propto \frac{1}{\text{compressibility}}$



$F = mg$ $\qquad$ $F = \frac{Mg}{2}$ $\qquad$ $F = mg + \frac{Mg}{2}$

$\Delta L = \frac{mgL}{YA}$ $\qquad$ $\Delta L = \frac{MgL}{2YA}$ $\qquad$ $\Delta L = \frac{\left(mg + \frac{Mg}{2}\right)L}{AY}$

The length of wire, when M_1 is Hung from it, is l_1 and l_2 with both Hanging. The natural length is

let natural length = l

$\Delta l = l_1 - l = \frac{m_1 g l}{AY}$ $\qquad$ $l_2 - l = \frac{(m_1 + m_2) g l}{AY}$

$$\frac{l_2 - l}{l_1 - l} = \frac{(m_1 + m_2) g l}{\cancel{AY}} \times \frac{\cancel{AY}}{m_1 g l} = \frac{m_1 + m_2}{m_1}$$

$$m_1 l_2 - m_1 l = m_1 l_1 - m_1 l + m_2 l_1 - m_2 l$$

$$m_1 (l_2 - l) = (m_1 + m_2)(l_1 - l)$$

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A, l, Y $\quad \Delta l = \frac{FL}{AY}$

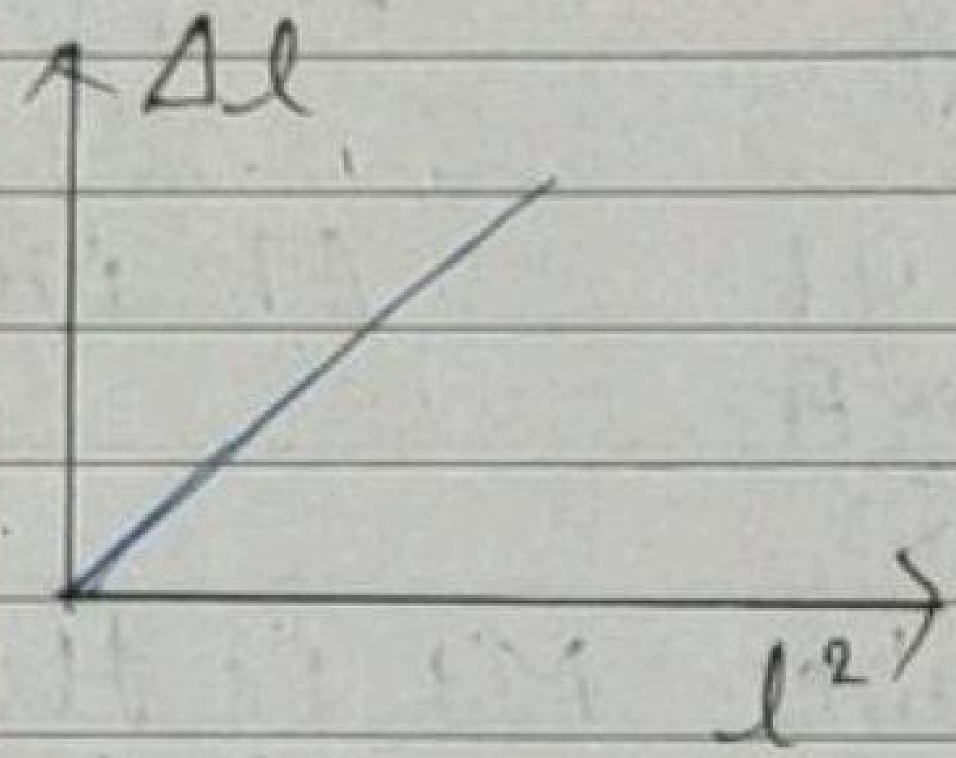
massless

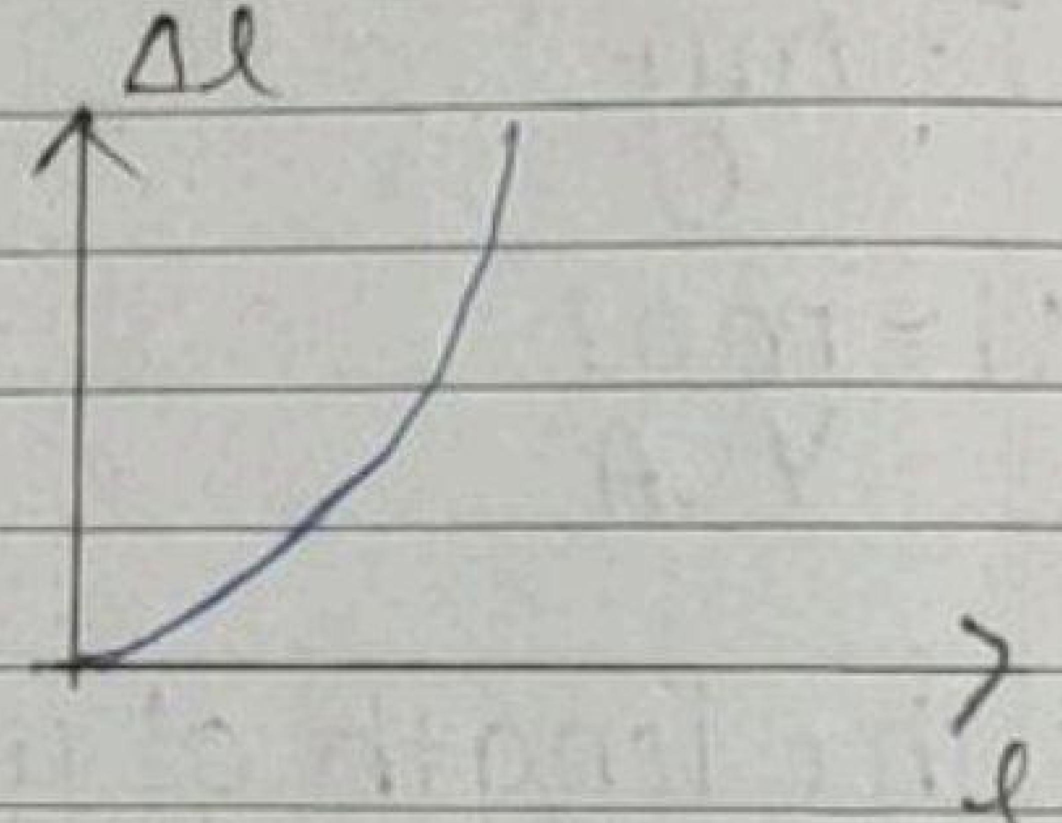
Fext

If volume is constant

$$\Delta l = \frac{FL \times L}{AY \times L} = \frac{FL^2}{VY}$$

$[\Delta l \propto L^2]$



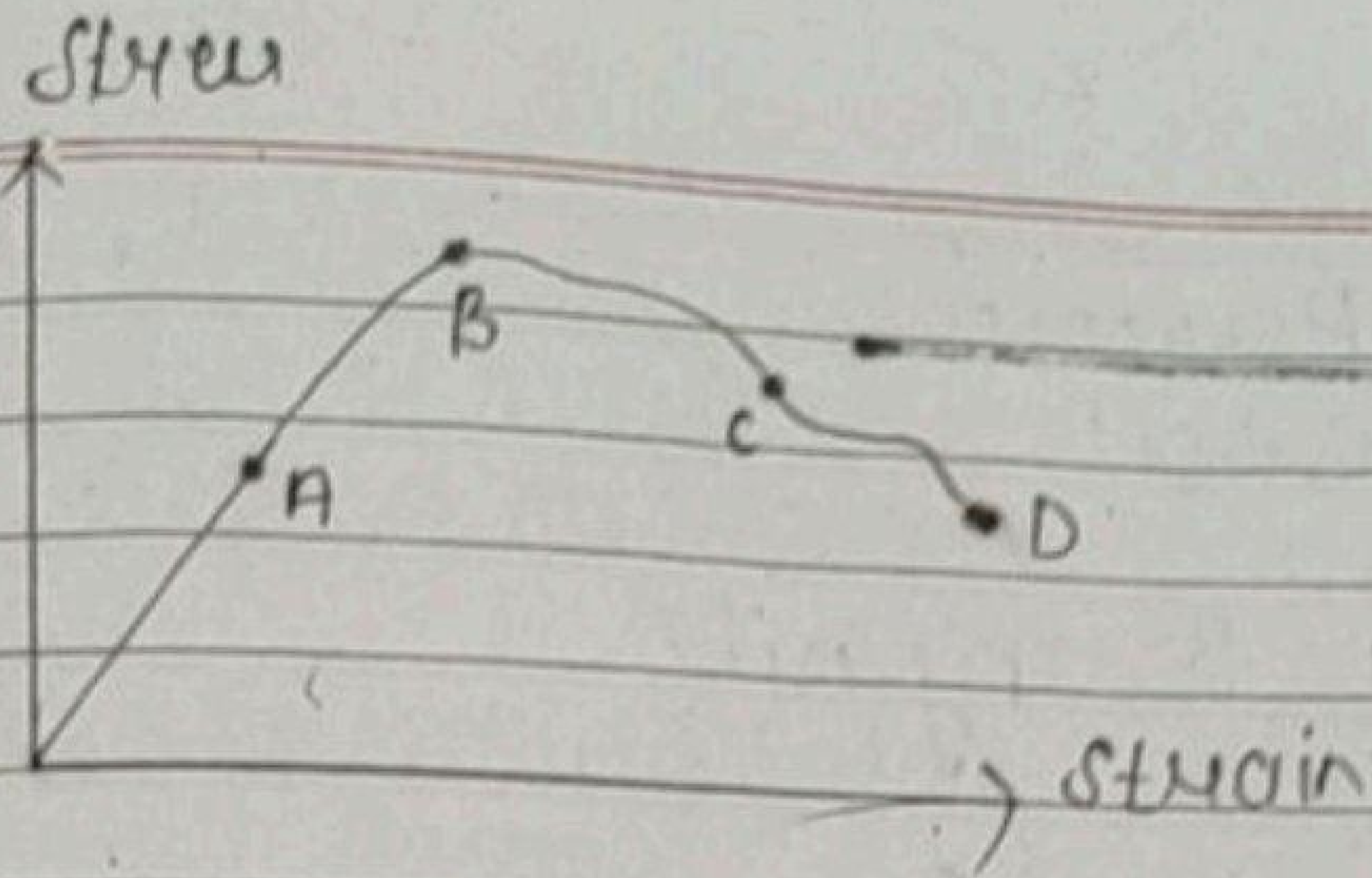


Q) The Bulk modulus of a spherical object is B. If it is subjected to uniform press. p. the fractional decrease in radius is

$\Delta p = -\left(\frac{\Delta V}{V} B\right)$ $\qquad = \frac{\Delta V}{V} = -\frac{\Delta P}{B}$

$V = \frac{4}{3}\pi r^3$ $\qquad 3\frac{\Delta r}{r} = -\frac{\Delta P}{B}$

$\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$ $\qquad \frac{\Delta r}{r} = -\frac{\Delta P}{3B}$



OA - Proportional limit
- Hooks law always Valid
- Conservative
- Completely regain its shape

OB - Elastic limit
- Hooks law not Valid
- Conservative after B non-conservative
- Completely regain its shape after B

CD - large
Ductile

CD - Small
Brittle
after B it partially regain its shape

Bulk modulus of gas in Different Process

1. for Isobaric Process
Pressure = Constn
$\Delta P = 0$
$\beta = 0$

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(2) **Isochoric Process-**

Volume = Const^n

$\Delta V = 0$

$B = \dfrac{\Delta P}{\left(\dfrac{\Delta V}{V}\right)} = \dfrac{1}{0}$ B = Infinite

(3) **Isothermal Process**

PV = Constant = T

$PV = C$

$\dfrac{\Delta P}{P} + \dfrac{\Delta V}{V} = 0$ $\dfrac{\Delta P}{P} = -\dfrac{\Delta V}{V}$

$B = \left(\dfrac{\Delta P}{-\dfrac{\Delta V}{V}}\right) = P$

(4) **Adiabatic Process**

Q = Constant

PV^γ = Const^n

$\dfrac{\Delta P}{P} + \gamma\dfrac{\Delta V}{V} = 0$ $\left(\dfrac{-\Delta P}{-\dfrac{\Delta V}{V}}\right) = \gamma P$

$B = \gamma P$

Breaking Stress-

Breaking force $\propto$ Area

Breaking force = σ Area
σ - Breaking stress
depend on nature of matter

$\sigma = \frac{\text{Breaking Force}}{\text{Area}}$ minimum length as with rod's $l = \frac{\sigma}{\rho g}$

Q) A force F is needed to break a copper wire of radius R. The force needed to break a copper wire of radius 2R will be.

Breaking Force $\propto$ Area
$\propto \pi r^2$

4F

Breaking Force : min^m force req. to break rod
Breaking stress : min^m stress at which rod will Break

Q) A rod is Having Vertical then find min^m length at which it will break due to its own weight.

$\sigma = \frac{\text{Breaking Force}}{\text{Area}}$ $l = ?$

σ Area = Breaking Force mg $\rho A l g = \sigma A$
$\sigma A = mg$
$\rho A l g = \sigma A$
$l = \left[\frac{\sigma}{\ldots}\right]$ Breaking stress

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Q) The breaking stress of aluminium is $7.5 \times 10^7\ Nm^{-2}$. The greatest length of aluminium wire that can hang vertically without breaking is $\rho = 2.7 \times 10^3\ kg\,m^{-3}$.

$$l = \frac{\sigma}{\rho g} = \frac{7.5 \times 10^7}{2.7 \times 10^3 \times 10} \quad 10^3$$

$$\frac{75 \times 10^3}{27} = 2.83 \times 10^3 m$$

Q) A wire can sustain a weight of 15 kg. If it cut into four equal parts, then each part can sustain a weight.

15 kg

Series Combination of wire

→ Tension same in both wire. stress com
→ Different elongation. (Δl) (Y)

$$\frac{F(l_1 + l_2)}{A Y_{eq}} = \frac{F l_1}{A Y_1} + \frac{F l_2}{A Y_2}$$

$$Y_{eq} = \left[\frac{l_1 + l_2}{\frac{l_1}{Y_1} + \frac{l_2}{Y_2}}\right] = \frac{2 Y_1 Y_2}{Y_1 + Y_2}$$

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Parallel Combination of rod-

Tension different σ (Y)

Same elongation ΔL

$$Y_{eq} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2} \quad \text{area same}$$

$$Y_{eq} = \frac{Y_1 + Y_2}{2}$$

Elastic Potential Energy

Store in a Body work $= F \cdot dx$

workdone by ext^n force in elongation dx will be

$dW_{ext} = F \cdot dx$

workdone by elastic force in elongation by dx is

$dW = F_{elastic} \, dx$

energy stored $=$ work $= \frac{AY(\Delta L)^2}{2L} = \frac{1}{2} F \Delta L$

Elastic Potential Energy-

$E = \frac{1}{2} F \Delta l \qquad \left[\frac{E}{AL}\right] = \frac{1}{2} \frac{F \Delta l}{AL}$

divide by AL

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Energy density $(u) = \frac{1}{2}\frac{F}{A}\frac{\Delta l}{l} = \frac{1}{2} \times \text{stress} \times \text{strain}$

$u = \frac{1}{2}\,\text{stress} \times \text{strain} = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}(\text{strain})^2 Y$

Q) If a rubber ball is taken at the depth of 200m in a pool, its volume decreases by 0.1%. if the density of water is $1 \times 10^3\ kg/m^3$ and $g = 10\ m/s^2$, then the volume elasticity in will be. ____

$\frac{\Delta V}{V} \times 100 = 0.1$ $\qquad \Delta P = B\frac{\Delta V}{V}$

$\frac{\Delta V}{V} = \frac{0.1}{100} = \frac{1}{1000} = 10^{-3}$ $\qquad B = \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{\rho g h}{10^{-3}}$

$B = 10^3 \times 10 \times 200 \times 10^3$
$= 2 \times 10^9$

Q) A 5m long wire is fixed to the ceiling. A weight of 10 kg is hung at the lower end and is m above the floor. The wire was elongated by 1 mm. The energy stored in the wire due to stretching is ____

$E = \frac{1}{2}F\Delta l = \frac{1}{2}mg \times 10^{-3} = \frac{1}{\cancel{2}} \cancel{10}^{5} \times 10 \times 10^{-3}$

$= 5 \times 10^{-2} = 0.05\,J$

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Q) When a block of mass M is suspended by a long wire of length L, the length of the wire becomes $(L+l)$. The elastic potential energy stored in the extended wire is.

$$E = \frac{1}{2} F \Delta l = \frac{1}{2} mgl$$

Q) If E is the energy stored per unit volume in a wire having young's modulus of the material Y, then stress is

$$E = \frac{1}{2} \frac{(\text{Stress})^2}{Y} \qquad \text{Stress} = \frac{\text{Stress}}{Y}$$

$$\sqrt{2EY} = \text{Stress}$$

Q) What is the percentage increase in length of a wire of diameter 2.5 mm, stretched by a force of 100 kg wt? $Y = 12.5 \times 10^{11}$

$$12.5 \times 10^{11} \frac{\text{dyne}}{\text{cm}^2} = \frac{12.5 \times 10^{11} \times 10^{-5}\,\text{N}}{10^{-4}\,\text{m}^2} = 12.5 \times 10^{11}$$

$$\frac{\Delta l}{l} \times 100 = \frac{F}{AY} \times 100 \qquad F\ 1\,\text{kg wt} = 10\,\text{N}$$

$$100 \times 10\,\text{N}$$

Q) A wire of 2m in length suspended vertically stretches by 10mm when mass of 10 kg is attached to lower end. The elastic potential energy gain by the wire.

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$U = \frac{1}{2} F \Delta l$ $\Delta l = 10 \times 10^{-3}$

$\frac{1}{\cancel{2}} \times \cancel{100}^{50} \times 10 \times 10^{-3}$

$500 \times 10^{-3} = 0.5 J$

Q) A wire suspended vertically from one end is stretched by attaching a weight 200N to the lower end. The weight stretchen the wire by 1mm. The elastic potential energy.

$U = \frac{1}{2} F \Delta l = \frac{1}{2} (\text{Stress} \times \text{Strain}) Al$

$\frac{1}{\cancel{2}} \cancel{200}^{100} \times 1 \times 10^{-3} = J = 0.1 J$

Q) The work done per unit volume to stretch the length of area of cross-section 2mm² by 2%. will be

$\frac{\Delta l}{l} \times 100 = 2\%$ $U = \frac{1}{2} (\text{Strain})^2 Y$

$\frac{\Delta l}{l} = \frac{2}{100}$ $= \frac{1}{\cancel{2}} \times \cancel{4} \times 10^{-4} \times 8 \times 10^{10}$

$\frac{\Delta l}{l} = 2 \times 10^{-2}$ $= 16 \times 10^{6}$ $16 m J/m^3$

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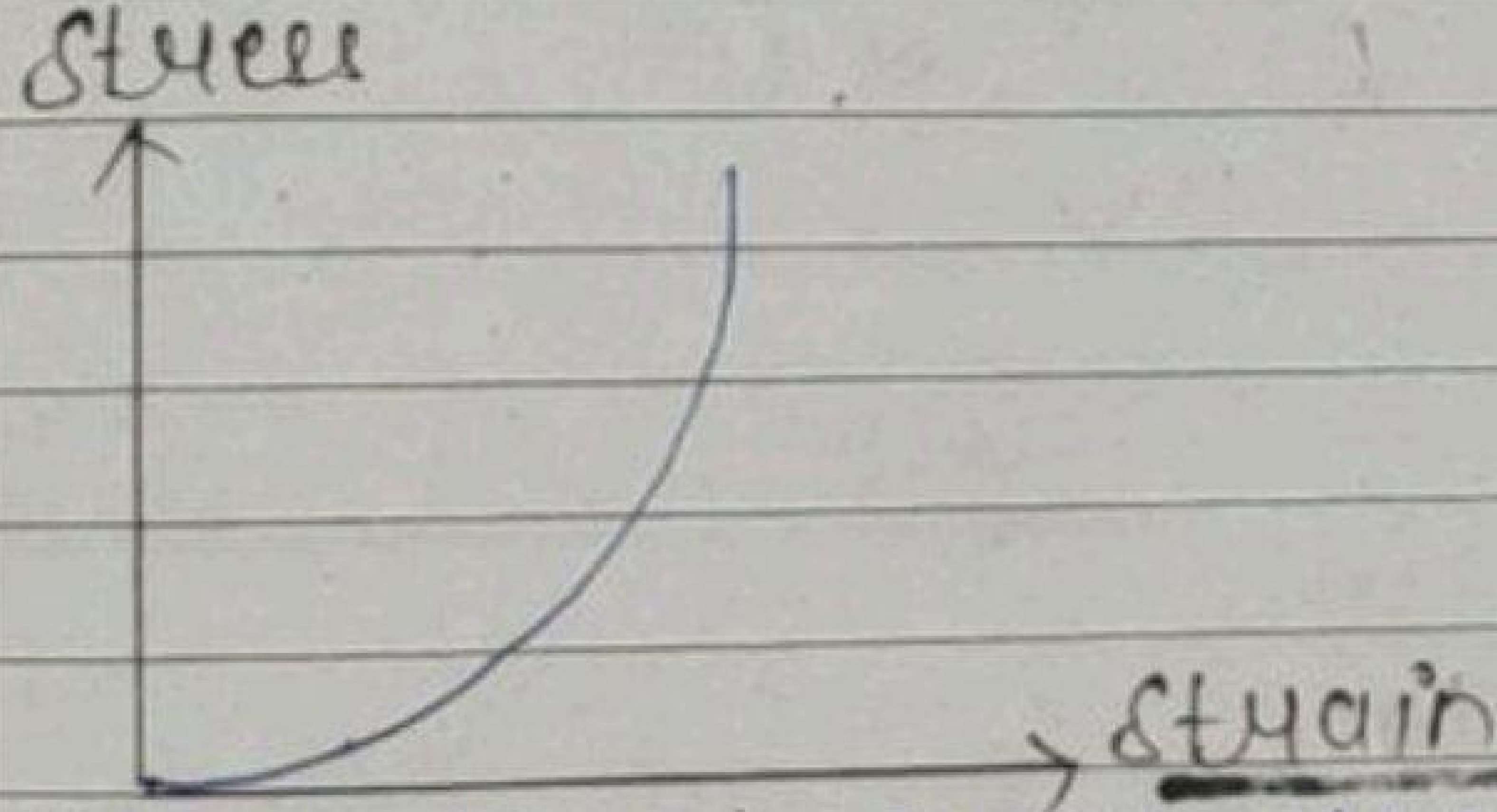
Q) A rod is suspend vertically then elastic energy stored in a rod due to own weight.

Batao - Batao



Stress - Strain curve for the elastic tissue of aorta, the large tube

Poissions ratio

$$\sigma = \frac{\text{Lateral Strain}}{\text{Longitudnal Strain}} = \frac{-\frac{\Delta R}{R}}{\frac{\Delta l}{l}}$$

$-1 \leq \sigma \leq 0.8$

if $\sigma = 0.5$ then Volume constant

B - Infinite

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Q) When a uniform metallic wire is stretched the lateral strain produced in it is β. If v and Y are poission's ratio and young's modulus for wire, then elastic potential energy density of wire.

$\frac{\Delta R}{R} = \beta$

$V = -\frac{\frac{\Delta R}{R}}{\frac{\Delta l}{l}} = -\frac{\beta}{\frac{\Delta l}{l}}$

$\frac{\Delta l}{l} = -\frac{\beta}{V}$

$u = \frac{1}{2}$ Strain × Stress

$u = \frac{1}{2}(\text{Strain})^2 \times Y$

$u = \frac{1}{2}\frac{\beta^2}{V^2} Y$

From Hooks law

Y - young's modulus

σ - Poissions ratio

$\boxed{\text{Stress} = -\frac{Y}{\sigma}}$

Q) The lengths of a metallic wire are L_1 and L_2 when the tension in the wire are T_1 and T_2 resp. Find the natural length of wire.

T_1 L_1

T_2 L_2

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$L_1 - L_0 = \frac{T_1 \cancel{L_0}}{\cancel{A}Y}$ $\quad T_2(L_1 - L_0) = T_1(L_2 - L_0)$

$L_2 - L_0 = \frac{T_2 \cancel{L_0}}{\cancel{A}Y}$ $\quad T_2L_1 - T_2L_0 = T_1L_2 - T_1L_0$

$T_2L_1 - T_1L_2 = T_2L_0 - T_1L_0$

T_2

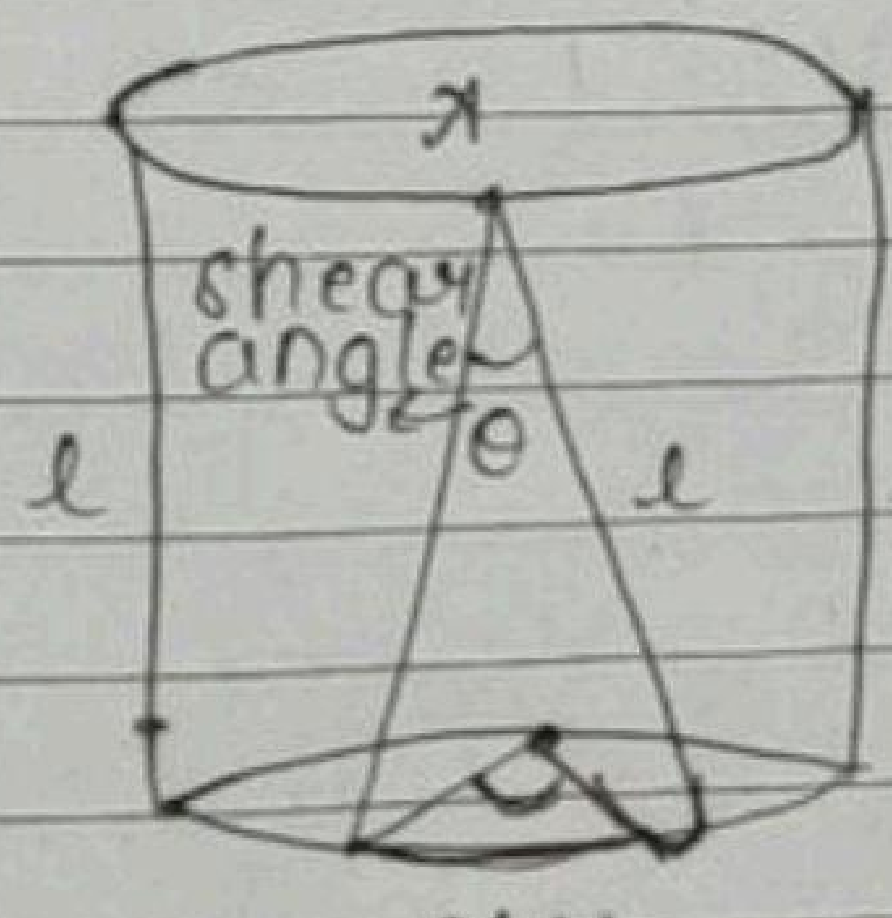
Q) The length of an elastic string in X m when the tension is 8N, and Y m when the tension is 10N. The length in m when the tension is 18N is

X m 8N $\quad x - l = \frac{8l}{AY} \quad \frac{x-l}{y-l} = \frac{\cancel{8}}{\cancel{10}} \frac{4}{5}$

Y m 10N

$y - l = \frac{10l}{AY}$

Twist Angle-



elastic torque

$\tau = C\alpha$ restoring torque

$C = \frac{\pi \eta r^4}{2l} \quad C \propto \frac{r^4}{l}$

$E = \frac{1}{2} C\alpha^2$

C - torsional constⁿ

arc $= l\theta$

arc $= R\alpha$

$l\theta = R\alpha$

θ - shear angle

α - twist angle